ECE 588

Fall 2018
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Homework 4

due Thursday, October 25, 2018

1. **[20 points]** Consider the following formulation of the basic production simulation problem.

   The equivalent load is defined to be

   $$\underset{\sim}{L}'_k = \underset{\sim}{L}'_{k-1} - \underset{\sim}{A}_k \qquad k = 1,2,...$$

   $$\underset{\sim}{L}'_o = \underset{\sim}{L}$$

   where $\underset{\sim}{L}$ is the load *r.v.* and $\underset{\sim}{A}_k$ is the availability *r.v.* of generating unit *k*.

   Let $\mathcal{L}'_k(x)$ be the *I.E.L.D.C*. for this formulation. **Derive** the relationship between $\mathcal{L}'_k(\bullet)$ and $\mathcal{L}_k(\bullet)$. **State** the convolution formula in terms of the $\mathcal{L}'_k(\bullet)$.

2. **[30 points]** Use the formulation of Problem 1 to **derive** the expressions for

   $\mathcal{U}_k$ = EUE after loading of unit *k*, $\qquad k = 0,1,2,...$

   $\mathcal{E}_k$ = expected generation of unit *k*, $k \geq 1$

   in terms of the $\mathcal{L}'_k(\bullet)$ s.

3. **[20 points] Verify** the expression

   $$\mathcal{L}_i(x) = \frac{1}{p_\alpha} \sum_{v=0}^{k-1} (-h)^v \mathcal{L}_{j-1}[x - vc_i] + (-h)^k \ , \qquad (k-1)c_i < x \leq kc_i \qquad (*)$$

   where,

$$\underset{\sim}{I} = \underset{\sim}{L} + \sum_{\substack{k=1 \\ k \neq i}}^{j-1} \underset{\sim}{Z}_k \quad ,$$

$\mathcal{L}_I(\cdot)$ is the *IELDC* corresponding to $\underset{\sim}{I}$,

and

$h = q_\alpha / p_\alpha$ with $p_\alpha + q_\alpha = 1$

4. **[20 points] Prove** using (*) that

$$\mathcal{L}_{j-1}(x) = p_\alpha \ \mathcal{L}_I(x) + q_\alpha \ \mathcal{L}_I(x - c_i)$$

5. **[20 points]** Consider the loading of an additional block of a unit α with one or more blocks already loaded. Let the unit cost of generation of this additional block be constant and equal to $\lambda\beta$. Assume that its expected generation is $\mathcal{E}\beta$. **Show** that the expected cost of generation is

$$C\beta = \lambda\beta \ \mathcal{E}\beta$$

6. **[20 points] Develop** the basic convolution formula and the expected energy relation for the case when unit i is a 3-state unit:

$$\underset{\sim}{A}_i = \begin{cases} c_i & \text{with probability } s_i \\ d_i & \text{with probability } r_i \\ 0 & \text{with probability } q_i \end{cases}$$

$$s_i + r_i + q_i = 1$$